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HEAT EXCHANGE IN A CIRCULAR PIPE IN MODELING OF TURBULENT AIR FLOW BY ORIENTED-FLUID FLOW

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Based on the anisotropic-turbulence model, the velocity profile and the temperature distribution in the flow in the case of steady-state air flow in a circular pipe have been obtained.

In the model of [1–3], a viscous fluid in the regime of turbulent flow in the wall region is considered as an oriented fluid, one kinematic parameter of which — the director — prescribes its local anisotropy. Within the framework of the model, the problem of determination of the velocity profile in the case of steady-state flow is formulated just as an ordinary boundary-value problem. A fairly good coincidence of the calculated velocity profiles in the case of confined flow between parallel planes [2] and in plane turbulent Couette flow [3] stimulates solution of other problems associated with turbulent flow.

In the present work, we investigate the problem on stationary heat exchange in the case of steady-state flow of a viscous incompressible fluid in a straight circular pipe at a constant wall temperature. First we consider the issues associated with finding the velocity profile and determining the parameters of a medium as functions of the parameters of flow; thereafter we approximately (by the Galerkin method) solve the equation of propagation of heat. The resulting regularities of variation in the local Nusselt number on the portion of stabilized heat exchange and on the initial thermal portion are compared to the experimental data for air.

Velocity Profile. A detailed solution of the problem (analogous to that considered in [2]) on determination of the velocity profile in confined flow between parallel planes enables us to restrict ourselves here only to a brief presentation.

Let a viscous incompressible fluid flow in the regime of steady-state turbulent flow in a straight circular pipe of radius R (the pipe is infinite and the walls are smooth). The direction of the x axis of the cylindrical coordinate system r, φ, x coincides with the direction of flow. Disregarding the mass forces, we find the velocity u_m and the director n_m in the form

$$u_x = u(r)\,, \quad u_r = u_\varphi = 0\,, \quad n_x = \cos\theta(r)\,, \quad n_r = \sin\theta(r)\,, \quad n_\varphi = 0\,. \tag{1}$$

The functions $u(r)$ and $\theta(r)$ sought must satisfy the equations [2]

$$\sin\theta\cos\theta\left(\theta'' + \frac{\theta'}{r}\right) - (2 - 3\cos^2\theta)\,\theta'^2 = 0\,, \tag{2}$$

$$\left(\mu_1 \sin^2\theta\cos^2\theta + \frac{\mu_4}{2}\right)u' = -\frac{r}{R}\tau_w \tag{3}$$

and the boundary conditions

$$\theta(R) = 0\,, \quad u(R) = 0\,. \tag{4}$$

The derivatives with respect to r in (2), (3), and in what follows are primed.

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Integrating Eq. (2) singly, we obtain

$$r \sin \theta \cos^2 \theta \theta' = - bR . \tag{5}$$

The "minus" sign and the factor R have been introduced into formula (5) for the sake of convenience. For the constant b to be different from zero, it is required that the derivative θ' become infinity on the pipe wall $r = R$. The value of b is determined by the condition at the upper boundary $r = r_0$ of the region in question with a vortex structure:

$$b = - (r_0/R) \sin \theta (r_0) \cos^2 \theta (r_0) \theta' (r_0) . \tag{6}$$

The angle θ grows with distance from the wall; therefore, we have $\theta'(r) < 0$ in the given coordinates and consequently $b > 0$. Integration of Eq. (5) with the first boundary condition of (4) yields the nonzero solution

$$\cos^3 \theta = 1 - 3b (R - r) \tag{7}$$

in addition to the zero solution.

Substituting expression (7) into Eq. (3) and integrating it with the second boundary condition of (4), we obtain the velocity profile sought in the form

$$\frac{u}{u_*} = A [F (t) - F (1)] , \tag{8}$$

$$F (t) = \frac{3bR - 1}{2\gamma^2 - 1} \left(\sqrt{\gamma^2 - 1} \arctan \frac{t}{\sqrt{\gamma^2 - 1}} + \frac{\gamma}{2} \ln \frac{\gamma - t}{\gamma + t} \right) + \frac{1 + 2\varepsilon}{4 (2\gamma^2 - 1)} \ln \frac{\gamma^2 - t^2}{t^2 + \gamma^2 - 1} + \frac{1}{4} \ln \left| t^4 - t^2 - \varepsilon \right| + \frac{t^2}{2} ,$$

$$A = \frac{\rho u_*}{3\mu_1 b^2 R} , \quad u_* = \left(\frac{\tau_w}{\rho} \right)^{1/2} , \quad t = \left[1 - 3bR \left(1 - \frac{r}{R} \right) \right]^{1/3} ,$$

$$\varepsilon = \frac{\mu_4}{2\mu_1} , \quad 2\gamma^2 = 1 + \sqrt{1 + 4\varepsilon} .$$

At short distances from the wall, formula (8), just as in [3], becomes the well-known logarithmic law of the wall with the same formulas for κ and C:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{(R - r) u_*}{\nu} + C , \tag{9}$$

$$\kappa = \frac{2\mu_1 b}{\rho u_*} , \quad C = \frac{1}{\kappa} \ln \frac{\nu b}{(\gamma - 1) u_*} .$$

Figure 1 gives experimental results [4] and curves calculated from formula (8) with the parameters corresponding to the experimental conditions (pipe diameter 246 mm) and the values of the quantities ε, μ_1, and b from Table 1 ("L" lines). The agreement of the results is rather good in the flow core ($\eta \geq 30$) up to the values allowed by the solution (8).

For the sake of comparison, the table also gives refined values of the quantities ε, μ_1, and b for which the calculated velocity profiles [2, 3] are in agreement with the experimental results of [5] ("CB" lines) and [6] ("ETR" lines). In all of the experiments [4–6], the working fluid is air, and the parameters ε, μ_1, and b in this work and in

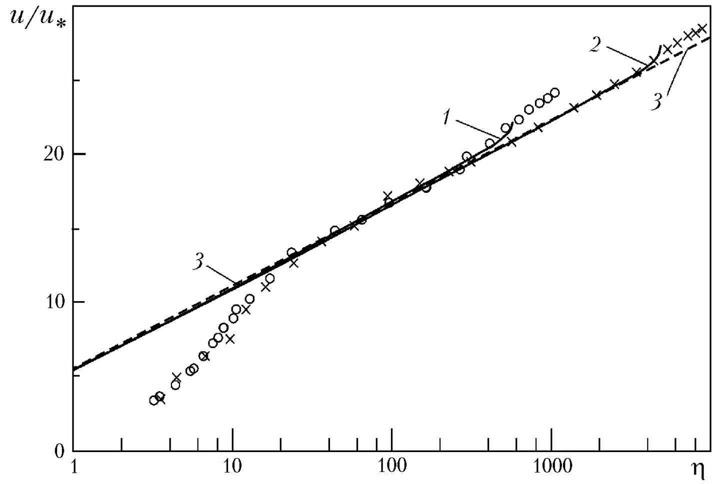

Fig. 1. Dimensionless velocity profiles u/u_* in the case of turbulent air flow in a circular pipe: 1 and 2) calculation from formulas (8); 3) logarithmic profile (9) for κ = 0.41 and C = 5.5 (1) Re = 40,260; 2 and 3) 429,200); points, experimental results [4].

TABLE 1. Values of the Parameters μ_1, ε, and b Ensuring Agreement between the Calculated [2, 3] and Experimental [4–6] Velocity Profiles

Experiments	Re	u_*, m/sec	μ_1, Pa·sec	$\varepsilon \cdot 10^5$	b, 1/m	μ_1/u_*, kg/m^2	$\mu_1\varepsilon \cdot 10^6$, Pa·sec
L	40 260	0.128	0.0061	16.4	4.93	0.048	1.00
L	429 200	1.08	0.052	1.86	5.00	0.048	0.97
CB	57 000	0.39	0.0185	4.20	4.72	0.047	0.78
CB	230 000	1.36	0.062	1.24	4.82	0.046	0.77
ETR	9500	0.293	0.0142	7.00	4.90	0.048	0.99
ETR	14 250	0.282	0.0135	7.40	4.83	0.048	1.00
ETR	12 640	0.363	0.0170	5.60	4.90	0.047	0.95

the solutions of [2, 3] have the same meaning; therefore, their comparison in different experiments is justified. It is clear from the table that the quantity b remains nearly constant in the case of both confined flow between the planes and plane Couette flow, and also in pipe flow. Furthermore, the quantities μ_1/u_* and $\mu_1\varepsilon$ remained virtually constant in these flows. Thus, it is assumed that, in motion of air near a solid wall, the equalities

$$b = \text{const}\,, \quad \mu_1 = m_1 u_*\,, \quad \mu_4 = 2\mu_1\varepsilon = \text{const} \tag{10}$$

hold true. Averaging the tabulated data, we have b = 4.83 m^{-1}, m_1 = 0.047 kg·m^{-2}, and μ_4 = 1.85·10^{-6} Pa·sec. Thus, the turbulent viscosity of air μ_t in pipe flow

$$\mu_t = \mu_1 \sin^2\theta \cos^2\theta + \frac{\mu_4}{2} \tag{11}$$

in the wall region is assumed to be quite determined.

The flow region in which the velocity profile may be described by formulas (8) is determined from (7) in the form of the inequality

$$3b\,(R - r) \le 1\,, \tag{12}$$

which corresponds to the experimental fact of existence of a stable structure of Λ vortices only in the wall region of flow [7]. Here we have $(R - r) \le 0.069$ m for b = 4.83 m^{-1}.

Heat Exchange. We find the temperature distribution in a semiinfinite pipe ($x \geq 0$) with a constant wall temperature in the case of steady-state fluid flow with the velocity profile (8). In the adopted coordinate system r, φ, x, the temperature T is found in the form

$$T = T(r, x) . \tag{13}$$

In view of the anisotropy of a moving fluid, which is locally determined by the director n_m, the heat-flux density q_m in cylindrical coordinates with condition (13) is prescribed by the equalities [8]

$$q_r = -(\lambda_0 + \lambda_1 n_r^2)\frac{\partial T}{\partial r} - \lambda_1 n_r n_x \frac{\partial T}{\partial x},$$

$$q_\varphi = -\lambda_1 n_\varphi \left(n_r \frac{\partial T}{\partial r} + n_x \frac{\partial T}{\partial x}\right), \tag{14}$$

$$q_x = -\lambda_1 n_x n_r \frac{\partial T}{\partial r} - (\lambda_0 + \lambda_1 n_x^2)\frac{\partial T}{\partial x}.$$

The coefficients λ_0 and λ_1 in a fixed regime of flow will be assumed to be constant. We replace the projection of n_m in (14) by their expressions (1) in terms of the angle θ. Substituting dependences (14) changed in such a manner for q_m into the general equation of propagation of heat for a moving medium in cylindrical coordinates [9, 10], we obtain

$$(\lambda_0 + \lambda_1 \sin^2\theta)\frac{\partial^2 T}{\partial r^2} + \left(\frac{\lambda_0 + \lambda_1 \sin^2\theta}{r} + \lambda_1 \sin 2\theta\theta'\right)\frac{\partial T}{\partial r} = \rho c_p u(r)\frac{\partial T}{\partial x} \tag{15}$$

with the boundary conditions

$$T(r, 0) = T_0 , \quad T(R, x) = T_{\text{w}} . \tag{16}$$

We introduce the dimensionless variables

$$\Theta = \frac{T - T_{\text{w}}}{T_0 - T_{\text{w}}}, \quad \xi = \frac{r}{R}, \quad X = \frac{x}{R}. \tag{17}$$

Substituting them and the velocity profile (8) into Eq. (15), we obtain

$$\frac{\partial^2\Theta}{\partial\xi^2} + \Psi_1(\xi)\frac{\partial\Theta}{\partial\xi} = \Psi_2(\xi)\frac{\partial\Theta}{\partial X}, \tag{18}$$

$$\Psi_1(\xi) = \frac{1}{\xi} - \frac{2\lambda_1 bR}{t(\xi)\,[\lambda_0 + \lambda_1(1 - t^2(\xi))]},$$

$$\Psi_2(\xi) = \frac{\rho c_p u_* AR\,(\Phi(\xi) - \Phi(1))}{\lambda_0 + \lambda_1(1 - t^2(\xi))},$$

$$t(\xi) = [1 - 3bR(1 - \xi)]^{1/3}, \quad \Phi(\xi) = F(t(\xi)) .$$

The boundary conditions in dimensionless variables are written as

$$\Theta(\xi, 0) = 1\,, \quad \Theta(1, X) = 0\,. \tag{19}$$

Since we cannot solve Eq. (18) analytically, we solve it approximately, by the Galerkin method [11], seeking the solution in the form

$$\Theta(\xi, X) = \sum_{k=1}^{n} g_k(X)\, \varphi_k(\xi)\,, \tag{20}$$

taking the functions

$$\varphi_k(\xi) = [\Phi_0(\xi) - \Phi_0(1)] \cos((k-1)\pi\xi)\,, \quad k = 1, 2, ..., n\,, \tag{21}$$

$$\Phi_0(\xi) = 2(3bR - 1) \ln \frac{\gamma - t(\xi)}{\gamma + t(\xi)} + \ln \frac{\gamma^2 - t^2(\xi)}{t^2(\xi) + \gamma^2 - 1} + \ln \left| t^4(\xi) - t^2(\xi) - \varepsilon \right| + 2t^2(\xi)$$

as the basis functions $\varphi_k(\xi)$ satisfying the second boundary condition of (19).

The functions $g_k(X)$ are the solutions of the system of differential equations

$$\frac{dg_i(X)}{dX} = \sum_{j=1}^{n} k_{ij} g_j(X)\,, \quad i = 1, 2, ..., n\,, \tag{22}$$

whose coefficients k_{ij} are the elements of the matrix K determined as

$$K = M^{-1} L\,, \quad L = (l_{ij})_1^n\,, \quad M = (m_{ij})_1^n\,, \tag{23}$$

$$l_{ij} = \int_0^1 \varphi_i(\xi) [\varphi_j''(\xi) + \Psi_1(\xi)\, \varphi_j'(\xi)]\, \xi d\xi\,,$$

$$m_{ij} = \int_0^1 \varphi_i(\xi)\, \varphi_j(\xi)\, \Psi_2(\xi)\, \xi d\xi\,.$$

Let h_i be the eigenvalues of the matrix K, $V = (v_{ij})_1^n$ be the matrix of its normalized eigenvectors (columns), and $P = (p_{ij})_1^n$ be the matrix with the elements

$$p_{ij} = \int_0^1 \varphi_i(\xi)\, \varphi_j(\xi)\, \xi d\xi\,.$$

Then the solution of system (22) with the first boundary condition of (19) satisfied by the Galerkin method has the form

$$g_k(X) = \sum_{i=1}^{n} M_i v_{ki} \exp(h_i X)\,, \quad M_i = \sum_{j=1}^{n} d_{ij} l_j\,, \tag{24}$$

where d_{ij} is the element of the matrix $D = (PV)^{-1}$ and $l_i = \int_0^1 \varphi_i(\xi) \xi d\xi$.

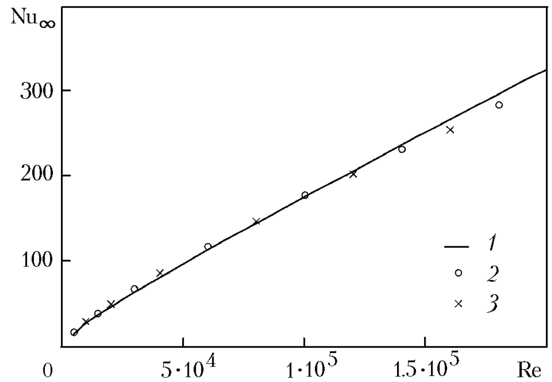

Fig. 2. Limiting local Nusselt number Nu_∞ vs. Reynolds number in air motion in a circular pipe: 1) calculation from (25); 2) (26); 3) (27).

Formulas (20), (21), and (24) with a prescribed n yield the approximate solution of the problem on temperature distribution in the flow. The local Nusselt number Nu may thereafter be computed from the formula [9, 10]

$$\mathrm{Nu} = -\frac{2}{\Theta}\left(\frac{\partial \Theta}{\partial \xi}\right)\bigg|_{\xi=1} . \tag{25}$$

To compare to experimental results we realized the solution under the specific conditions formulated above: the fluid is a viscous gas (ρ = 1.205 kg/m^3, $\nu = 1.5\cdot10^{-5}$ m^2/sec, $\lambda = 2.57\cdot10^{-2}$ W/(m·K), c_p = 1002 J/(kg·K), and Pr = 0.705); the pipe walls are smooth. The calculated pipe radii R = 30, 40, 50, 60, and 69 mm were selected so that the velocity profile (8) filled the entire cross section of the pipe. The coefficients of the model were in agreement with the experimental data: $\mu_1 = 0.047u_*$, $\mu_4 = 1.85\cdot10^{-6}$ Pa·sec, $\lambda_1 = 46.5u_*$, and $\lambda_0 = k_0u_*$, where k_0 = 0.0084, 0.0112, 0.0141, 0.0169, and 0.0195 W·sec/(m^2·K) respectively for the radii R = 30, 40, 50, 60, and 69 mm. The calculations were carried out for n = 25.

Figure 2 gives the limiting Nusselt number Nu_∞ computed from formula (25) for X = 200 as a function of the Reynolds number Re (curve). The calculations have shown that the values of Nu for the same Re virtually do not differ for all the pipe radii considered; therefore, Fig. 2 only gives the plot for R = 50 mm. For the sake of comparison, the figure gives the points found from the empirical formulas [12]

$$\mathrm{Nu}_\infty = 0.021\mathrm{Re}^{0.8}\mathrm{Pr}^{0.43} \tag{26}$$

and by Petukhov et al. [10]

$$\mathrm{Nu}_\infty = \frac{\frac{f}{8}\,\mathrm{Re\,Pr}}{1 + \frac{900}{\mathrm{Re}} + 12.7\sqrt{\frac{f}{8}}\left(\mathrm{Pr}^{2/3} - 1\right)}, \quad f = \frac{1}{(1.82\log\mathrm{Re} - 1.64)^2} . \tag{27}$$

Since the empirical formulas were obtained in heat exchange with a constant heat-flux density on the wall, the results of computations from them were corrected for heat exchange with a constant wall temperature according to [10]. In the range $10^4 \le \mathrm{Re} \le 1.5\cdot10^5$, the deviation of the calculated curve from the empirical points does not exceed 4%; for $\mathrm{Re} > 1.5\cdot10^5$ it is already considerable.

Figure 3 compares the ratio Nu/Nu_∞ obtained by calculation from formula (25) for Re = 50,000 and R = 50 mm on the initial thermal portion to the empirical formulas of [13] and [10], respectively:

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_\infty} = 3.45\mathrm{Re}^{-0.1}\left(\frac{x}{d}\right)^{-0.22}\cdot 10^{0.1\sqrt{[\log(\mathrm{Re}x/d) - 3.9]^2 + 0.01}} , \tag{28}$$

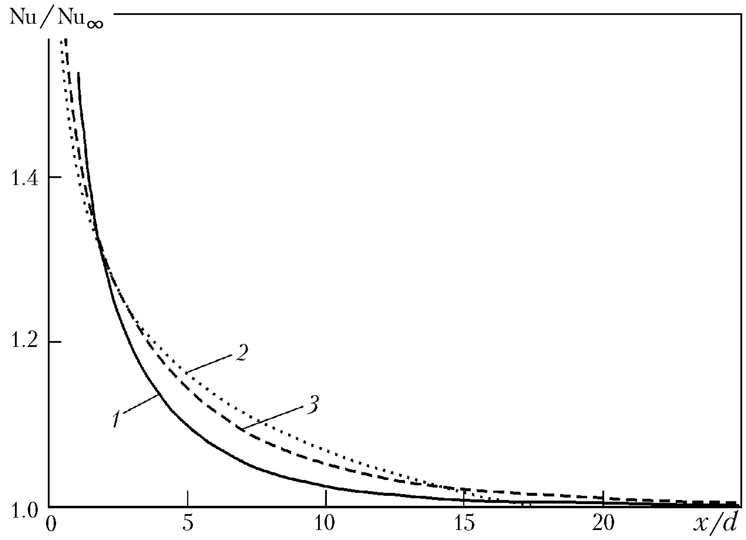

Fig. 3. Change in the local Nusselt number on the initial thermal portion of a circular pipe as a function of *x/d*: 1) calculation from (25); 2) (28); 3) (29).

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_\infty} = 1 + 0.48 \left(\frac{x}{d}\right)^{-\frac{1}{4}} \left(1 + \frac{3600}{\mathrm{Re}} \sqrt{\frac{d}{x}}\right) \exp\left(-0.17\,\frac{x}{d}\right). \tag{29}$$

The coincidence of the results may be assumed to be satisfactory, with this solution yielding a length of the initial thermal portion virtually coincident with the experimental length.

Thus, the employed wall-turbulence model constructed based on the local anisotropy of turbulent flow quite adequately describes both the velocity profile and the temperature profile in pipe flow. Both profiles are determined within the framework of one model. Experimental data are used only to prescribe the properties of the medium.

NOTATION

A, coefficient in (8); a, thermal diffusivity, m^2/sec; b, integration constant in (5), 1/m; C, constant of the wall's law in (9); c_p, specific heat at constant pressure, J/(kg·K); d, pipe diameter, m; f, coefficient of resistance of the pipe; $\mathrm{Nu} = \alpha d/\lambda$, Nusselt number; n, number of basis functions in the Galerkin method; n_m, director; $\mathrm{Pr} = \nu/a$, Prandtl number; q_m, heat-flux density, W/m^2; R, pipe radius, m; $\mathrm{Re} = wd/\nu$, Reynolds number; r, φ, x, cylindrical coordinates; T, local temperature, K; T_0 and T_w, temperatures determined by conditions (16) at the boundary of the flow, K; u_m, local velocity, m/sec; u, longitudinal local velocity, m/sec; u_*, dynamic velocity, m/sec; w, mean velocity, m/sec; X, dimensionless longitudinal coordinate; α, heat-transfer coefficient, W/(m^2·K); γ, constant; $\varepsilon = \mu_4/(2\mu_1)$; $\eta = u_*(R - r)/\nu$, dimensionless distance from the wall; Θ, temperature (17); Θ, dimensionless mean mass temperature; θ, angle between the director and the x axis; κ, Kármán constant; λ, molecular thermal conductivity, W/(m·K); λ_0 and λ_1, anisotropic turbulent thermal conductivities, W/(m·K); μ_1 and μ_4, coefficients of anisotropic turbulent viscosity, Pa·sec; ν, kinematic viscosity, m^2/sec; ξ, dimensionless coordinate; ρ, density, kg/m^3; τ_w, modulus of tangential stress on the wall, Pa. Subscripts: m, coordinates r, Φ, x; i, j, and k, natural numbers; w, wall.

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